Funky numbers in physics

Of all the ideas that the great Hungarian-American physicist Eugene Wigner contributed to science, perhaps the most intriguing is his phrase "the unreasonable effectiveness of mathematics". In a 1960 essay, Wigner called its appropriateness for the natural sciences a "miracle". It is, at the very least, uncanny. What gives a handful of integrals the right to predict what an experimentalist measures for, say, the electron's anomalous magnetic moment? Julian Schwinger, who used his version of quantum electrodynamics to calculate the first-order correction to this quantity, seems to have appreciated the oddness of the situation: he has "$\alpha/2\pi$" above his name on his tombstone. (His wife's name is below.)

But some mathematics is more unreasonable than others. In 1989 the British physicist Paul Davies found that a spinning black hole transitions from an object with negative specific heat – yes, it gets cooler as it gains energy – to a positive specific heat at the point where $J^2/M^4 = 1/\phi$, where J is its angular momentum, M its mass and ϕ the so-called "golden ratio", $\phi = (1+\sqrt{5})/2 \approx 1.618$. What in the universe does this ratio – heralded by mathematicians, artists and architects since Euclid as the most aesthetically pleasing proportion for a rectangle – have to do with black holes?

Alas, a graduate student named Cesar Uliana recently found an error in Davies' work. The true ratio now seems to be a no-less-baffling $2\sqrt{3}-3 \approx 0.464$. However, the golden ratio does still appear in the special case where the black hole's internal energy increases while J/M^2 is held constant. And – showing that a good number can't be kept down – a team of German and British researchers investigating Ising-like chains of the magnetic material cobalt niobate found that certain of its resonant energy levels have a ratio equal to the "golden" one, an intriguing suggestion of far deeper symmetries somewhere in the physics.

Of course, no number in physics is more bedevilling than 1/137, which some physicists of the last century considered near holy writ, since it is tantalizingly close to the dimensionless "fine-structure constant" $\alpha = e^2/\hbar \approx 1/137.035999074$. Max Born summed up the views of many when, during a job-seeking trip to India in 1935 after the Nazis ran him out of Göttingen, he declared "A perfect theory should be able to derive the number α by a purely mathematical reasoning without recourse to experience." Such a result would mean one of e, $\hbar$ or c is not independent of the other two, and all true physicists would give their right hand to derive such a result – plus a few fingers from their left.

Before α's reciprocal was known not to be an integer, Arthur Eddington believed it to be 136, which he claimed to derive from the idea of a 4D Dirac electron as $4^2(4^2+1)/2$. (He also claimed there were exactly 136×2^{256} protons in the universe.) When α was later found to be closer to 1/137, Eddington found an excuse to add unity to his earlier result, whereupon *Punch* magazine labelled him "Sir Arthur Adding-One."

Eddington died with all 10 of his fingers, and the fine-structure constant intrigues physicists still. Richard Feynman said "All good theoretical physicists put this number up on their wall and worry about it." In 1975 two good theoretical physicists, Stanley Deser and Peter van Nieuwenhuizen (then at Brandeis University), were trying to find a quantum theory of general relativity. When they combined it with Maxwell's equations, they found a term that, if added to the basic theory, would make it finite at the one-loop level of its perturbation expansion. The term, calculated from a thicket of Maxwell and Einstein tensors, had a proportionality constant of 137/60. Van Nieuwenhuizen, who went on to co-discover supergravity, called the result "curious", at which God presumably had a good chuckle. Gravity hasn't been quantized to this day.

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My friend Joe Milana, who shared an adviser with me in graduate school, has his own favourite example of funky numbers in physics: π^2-9. This curious factor appears in the calculation of the lifetime of the triplet state of positronium (the spin-opposite bound state of an electron and positron) and charmonium. Because the factor appears in the denominator, it boosts the decay time by a factor of about 10 – an awfully clever way for nature to make physicists work hard for their understanding. Joe also once wrote a paper about hadron–hadron scattering in which the number e^e made an appearance. I seem to have taken more joy from it than he has, and if I outlive him I may put it on a sticky note on his tombstone.

"I have never done anything 'useful'," wrote the number theorist G H Hardy in *A Mathematician's Apology*. "No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world." But these days even number theory is being applied, whether to the task of better cryptography or to connections between quantum energy levels in large nuclei and the Riemann zeta function, a complex function involved in the distribution of prime numbers. Somewhere late tonight, a number theorist may develop mathematics that is just what a 22nd century physicist is looking for. As Wigner wrote, "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."

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